JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 43, No. 2, March–April 2006

Notable Two-Synodic-Period Earth–Mars Cycler

T. Troy McConaghy,* Damon F. Landau,† Chit Hong Yam,‡ and James M. Longuski§
Purdue University, West Lafayette, Indiana 47907-2023

Earth–Mars cycler trajectories (cyclers) could play an important role in a future human transportation system to Mars. A particular cycler that repeats every two synodic periods and has one intermediate Earth encounter is very promising. In a circular-coplanar model it requires no propulsive maneuvers, has 153-day transfer times between Earth and Mars, and has arrival V_∞ magnitudes of 4.7 km/s at Earth and 5.0 km/s at Mars. A method to find an analog cycler in a more realistic model (i.e., using an accurate ephemeris for the states of Earth and Mars) is described. Two cost metrics are considered: total cycler ΔV and total cycler ΔV plus total taxi ΔV. Numerical solutions are presented for both metrics. The total required ΔV is very small, though not zero. If the Earth–Mars and Mars–Earth transit times are constrained, then the characteristics of the optimal cycler trajectory change. Tradeoffs between maximum transit time and other mission characteristics are analyzed for all possible launch periods.

Nomenclature

$\boldsymbol{a}$	=	orbital elements vector
$\boldsymbol{g}$	=	vector of V_∞ constraints, km/s
h	=	flyby altitude, km
$\boldsymbol{h}$	=	vector of flyby altitude constraints, km
N	=	number of intermediate Earth flybys
$\boldsymbol{P}$	=	primer vector
$\boldsymbol{R}$	=	position vector of the cycler vehicle, AU
S	=	Earth–Mars synodic period, year
T	=	time for orbits of Earth and Mars to repeat inertially, year
t	=	time since launch, year
V_∞	=	hyperbolic excess speed, km/s
ΔV	=	magnitude of change in velocity, km/s
τ	=	duration of leg 1, 2.8276 years

Subscripts

i	=	Earth encounter index
in	=	incoming
out	=	outgoing

Introduction

A FUTURE Earth–Mars transportation system will probably use several kinds of spacecraft trajectories. Whereas some trajectories are well suited for human transportation, others are better suited for ferrying supplies. To develop an efficient system, mission designers should know what trajectories are available.

One potentially useful type of trajectory is the Earth–Mars cycler trajectory, or cycler. A spacecraft on a cycler, the "cycler vehicle," regularly passes close to both Earth and Mars (but never stops at either). A "taxi vehicle" transfers people between the cycler vehicle and Earth (or between the cycler vehicle and Mars). Cyclers that require propulsive maneuvers are referred to as "powered cyclers," whereas cyclers that rely on gravitational forces only are called "ballistic cyclers." Although not considered here, there are some variations on cyclers, in which the spacecraft enters a temporary parking orbit at Mars (Mars–Earth semicyclers),[1,2] at Earth (Earth–Mars semicyclers),[3] or at both Mars and Earth (stop-over cyclers).[4,5]

It appears that Rall was the first to systematically investigate Earth–Mars cyclers, for his doctoral thesis in 1969 (Ref. 6). This research was later summarized by Rall and Hollister.[7] The cycler idea languished until 1985, when Aldrin first proposed the "Aldrin cycler"[8] and Niehoff first proposed the VISIT 1 and VISIT 2 cyclers.[9–11] The existence of the Aldrin cycler was verified by Byrnes et al.,[12] and the Aldrin cycler was compared to the VISIT cyclers by Friedlander et al.[13]

In 2002, McConaghy et al.[14] showed that the Aldrin cycler and the VISIT cyclers are special cases in a larger family of cyclers. Several new cyclers were also identified by Byrnes et al.,[15] Chen et al.,[16–18] McConaghy et al.,[19,20] Rauwolf et al.,[21] and Russell and Ocampo.[22–24]

Among the recently discovered cyclers, one is particularly noteworthy. Known as the "ballistic S1L1 cycler"[14] (for reasons to be explained later), it has relatively short Earth–Mars (or Mars–Earth) transfer times and low V_∞ at both Earth and Mars (see Fig. 1). The ballistic S1L1 cycler was first identified in 2002 (Ref. 14). It is similar to the "case 3" cycler of Byrnes et al.[15] Using the cycler nomenclature proposed in Ref. 25, it has the cycler label "2g(2.8276, 658°, U)g(1.4580, 525°, L)." Further analysis of the ballistic S1L1 cycler appeared in McConaghy et al.,[19,20] but some questions still remained. In this paper, we give the most thorough analysis to date.

We begin with an analysis of the ballistic S1L1 cycler in a simplified, circular-coplanar model of the solar system. A process for finding and optimizing cycler trajectories in an ephemeris-based model is then described, and numerical results are summarized.

Circular-Coplanar Analysis

In our analysis of the ballistic S1L1 cycler, we make the following simplifying assumptions (some of which will be relaxed later):

1) The Earth–Mars synodic period is exactly $2\frac{1}{7}$ years.

2) Earth's orbit, Mars's orbit, and the cycler trajectory lie in the ecliptic plane.

3) Earth and Mars have circular orbits.

4) The cycler trajectory is conic, that is, it is a solution to the sun-spacecraft two-body problem.

5) Only the Earth has sufficient mass to provide gravity-assist maneuvers. (Mars can be encountered on a leg, but the encounter does not change the orbit.)

6) Gravity-assist maneuvers occur instantaneously.

Received 21 December 2004; revision received 29 April 2005; accepted for publication 9 May 2005.

*Ph.D. Candidate, School of Aeronautics and Astronautics; troy_mcconaghy@hotmail.com. Student Member AIAA.

†Ph.D. Candidate, School of Aeronautics and Astronautics; nutmunky@purdue.edu. Student Member AIAA.

‡Graduate Student, School of Aeronautics and Astronautics; cyam@purdue.edu. Student Member AIAA.

§Professor, School of Aeronautics and Astronautics; longuski@ecn.purdue.edu. Associate Fellow AIAA.

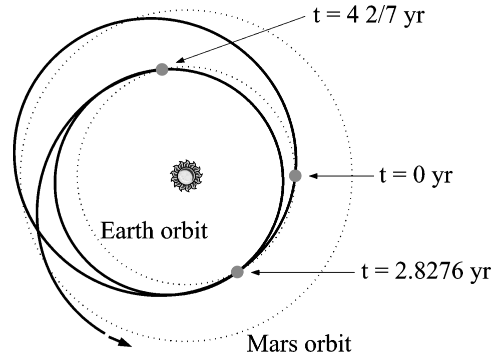


Fig. 1 Ballistic S1L1 cycler in a simplified solar-system model.

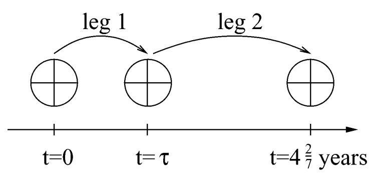


Fig. 2 Assumed timing of the Earth encounters.

We note that assumption 1 is equivalent to assuming that the orbital period of Mars is 1.875 years (whereas a more accurate value is 1.881 years). Also, assumption 1 implies that the semimajor axis of Mars is $1.875^{2/3}$ astronomical units (AU) ≈ 1.5206 AU.

For a trajectory to be an Earth–Mars cycler, it must return to Earth after an integer number of synodic periods. The ballistic S1L1 cycler returns to the Earth after two synodic periods ($4\frac{2}{7}$ years), but there is also an intermediate Earth encounter that happens at a time $\tau = 2.8276$ years after the initial Earth departure. The timing of the Earth encounters during a two-synodic-period time interval is illustrated schematically in Fig. 2.

Because we assume that the orbits of Earth and Mars are circular, we can choose a Cartesian coordinate system so that the position vector of the Earth at time $t = 0$ is $[1, 0]$. Because the angular velocity of the Earth is 2π radians per year, the position of the Earth at any later time t will be $\boldsymbol{R}_{\text{Earth}}(t) = [\cos(2\pi t), \sin(2\pi t)]$. Because the spacecraft encounters the Earth at times $t = 0$, τ, and $4\frac{2}{7}$, the position vectors of the spacecraft at the first, second, and third Earth encounters are, respectively,

$$\boldsymbol{R}_1 = [1, 0], \qquad \boldsymbol{R}_2 = [\cos(2\pi\tau), \sin(2\pi\tau)] \approx [0.4690, -0.8832]$$

$$\boldsymbol{R}_3 = \left[\cos\left(2\pi \cdot 4\tfrac{2}{7}\right) \sin\left(2\pi \cdot 4\tfrac{2}{7}\right)\right] \approx [-0.2225, 0.9749]$$

Finding the cycler trajectory connecting $\boldsymbol{R}_1$ to $\boldsymbol{R}_2$ in a time τ is a Lambert problem.[26] It turns out that there are multiple solutions. Indeed, one of the solutions is the orbit of the Earth (which is not what we want). Among the solutions, there are two that make between one and two revolutions of the sun. The ballistic S1L1 cycler uses the solution with the shortest period: 1.4889 years (so the semimajor axis is 1.3039 AU). (Other solutions were tried, but they led to different cyclers that have undesirable characteristics.)

Finding leg 2 [connecting $\boldsymbol{R}_2$ to $\boldsymbol{R}_3$ in $4(\frac{2}{7} - \tau)$ years] is also a Lambert problem. As with leg 1, there are two solutions that make between one and two revolutions of the sun. The ballistic S1L1 cycler uses the solution with the longest period: 1.0733 years (so the semimajor axis is 1.0483 AU). (The name "S1L1 cycler" is a reminder that the first leg is the short-period solution making 1–2 revs, and the second leg is the long-period solution making 1–2 revs.)

When we calculate the incoming and outgoing V_∞ at Earth, we find that they are both equal to 4.7128 km/s. This equality is not a coincidence; the duration of leg 1 is chosen to make the incoming and outgoing V_∞ equal so that the required velocity change can be accomplished using a gravity-assist flyby. No ΔV maneuvers are required to keep the spacecraft on its orbit, hence the name "ballistic S1L1 cycler."

Figure 1 shows the orbits of Earth, Mars, and the ballistic S1L1 cycler (in the simplified, circular-coplanar solar-system model). (The orbit of the cycler is shown for the first five years: enough time to complete one two-synodic-period cycle and start the next cycle.) The cycler vehicle starts at Earth at time $t = 0$ years. It then makes almost two revs of the sun before returning to Earth at time $t = 2.8276$ years. A gravity-assist flyby at an altitude of 31,809 km transfers the cycler vehicle onto the smaller orbit of leg 2. After another 1.5 solar revolutions (approximately), the spacecraft returns to Earth again (at time $t = 4\frac{2}{7}$ years). A gravity-assist maneuver (also at an altitude of 31,809 km) boosts the spacecraft back onto the larger orbit of leg 1 and the $4\frac{2}{7}$-year cycle begins anew.

We note that the cycler vehicle crosses the orbit of Mars on leg 1 but not on leg 2. (The aphelion of the first leg is 1.6369 AU, whereas the aphelion of the second leg is 1.2170 AU. We recall that Mars is assumed to have a semimajor axis of 1.5206 AU.) Because leg 1 makes almost two solar revs, it crosses the orbit of Mars four times: when $t = 0.4193$, 0.9194, 1.9082, and 2.4083 years (i.e., 153.15, 335.81, 696.97, and 879.63 days). The transfer time from Earth at $t = 0$ to the first Mars crossing (0.4193 years) is the same as the transfer time from the fourth Mars crossing to Earth at $t = 2.8276$ ($2.8276 - 2.4083 = 0.4193$ years). The reason for the equality can be seen in Fig. 1. The leg 1 line of apsides runs midway between Earth at $t = 0$ and Earth at $t = 2.8276$. Consequently, the transfer arc from Earth at $t = 0$ to the first Mars-orbit crossing is the mirror image of the transfer arc from the fourth Mars-orbit crossing to Earth at $t = 2.8276$.

For the spacecraft to encounter Mars during one of the Mars-orbit crossings, an appropriate launch date must be chosen. For example, one can choose the launch date so that the spacecraft will encounter Mars 153.15 days after launch (but not at the other three Mars-orbit crossing times). If the launch date is chosen so that there is a short-duration Earth–Mars transfer (153.15 days in the circular-coplanar model), then we call the cycler an "outbound cycler" (because it can be used to transport humans from Earth out to Mars). Similarly, one can choose the launch date so that there is a short-duration (153.15-day) Mars–Earth transfer (between the fourth Mars-orbit crossing and the Earth encounter at time $t = 2.8276$ years). Such a cycler is called an "inbound cycler." By calculating transfer angles and using phasing analysis, one can show that the minimum time between the launch of an inbound cycler vehicle and the launch of an outbound cycler vehicle is 0.3336 years.

If we assume that humans can only tolerate short transfer times (of 180 days or less), then at least two cycler vehicles are needed, one outbound and one inbound. Figure 3 illustrates the timeline of a two-vehicle system. Because Earth and Mars (are assumed to) have circular orbits, their distance from the sun is constant. The times where one of the vehicles encounters Earth or Mars are indicated by dots. The single outbound vehicle provides an outbound transfer once every $4\frac{2}{7}$ years, and the single inbound vehicle provides an inbound transfer once every $4\frac{2}{7}$ years. In Fig. 3, we have chosen to set $t = 0$ when the inbound cycler launches. Consequently, $t = 0.3336$ years when outbound cycler launches, and $t = 0.3336 + 0.4193 = 0.7529$ years when the outbound cycler encounters Mars. Because the inbound cycler encounters Mars at

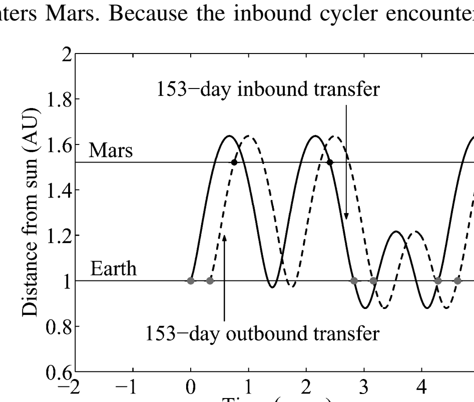


Fig. 3 Two-vehicle system providing short transfers.

Table 1 Ballistic S1L1 cycler characteristics (from circular-coplanar analysis)

Characteristic	Value
ΔV required	0 km/s
Earth V_∞	4.7 km/s
Earth flyby altitude	31,809 km
Mars V_∞	5.0 km/s
Mars flyby altitude	∞[a]
Earth–Mars transfer time	153.15 days
Minimum Mars stay time	604.63 days
Mars–Earth transfer time	153.15 days
Repeat time	$4\frac{2}{7}$ yr

[a]Gravity assist not required from Mars.

time $t = 2.4083$ years, the minimum time between a Mars arrival (by the outbound vehicle) and a Mars departure (by the inbound vehicle) is $2.4083 - 0.7529 = 1.6554$ years (or 604.63 days). Consequently, a minimum-duration trip from Earth to Mars to Earth (with a stay at Mars) lasts $153.15 + 604.63 + 153.15 = 910.93$ days (or 2.4940 years). If the Mars rendezvous is aborted, then the crew will return to Earth after 2.8276 years.

Because the Earth–Mars synodic period is assumed to be $2\frac{1}{7}$ years, there is a launch period for an outbound vehicle every $2\frac{1}{7}$ years. Similarly, there is a launch period for an inbound vehicle every $2\frac{1}{7}$ years. By using two outbound vehicles with launch dates $2\frac{1}{7}$ years apart and two inbound vehicles with launch dates $2\frac{1}{7}$ years apart, short Earth–Mars and Mars–Earth transfers would be available every $2\frac{1}{7}$ years. When using four vehicles, the minimum Mars stay time and minimum round-trip time are the same as when using two vehicles.

Table 1 summarizes some pertinent characteristics of the ballistic S1L1 cycler. We note that the V_∞ at Earth and Mars are significantly lower than the corresponding V_∞ for the Aldrin cycler (6.5 and 9.8 km/s, respectively).[12] Such a reduction in V_∞ translates into a reduction in the propellant required by the taxi vehicles.

Analysis Using an Ephemeris Model of the Solar System

After finding the ballistic S1L1 cycler in our simplified (circular-coplanar) model of the solar system, we wanted to know whether it exists and is still ballistic in an ephemeris model. By an ephemeris model, we mean that the positions and velocities of the planets are obtained from an integrated ephemeris, such as the Jet Propulsion Laboratory's (JPL's) DE405. We continue to ignore third-body gravitational effects, except during the planetary flybys. Gravity-assist maneuvers can occur at both Earth and Mars but are still modeled as an instantaneous rotation of the V_∞ vector.

To show that a cycler is ballistic in the ephemeris model, we would have to propagate it for an infinite time. Because that is impossible, we instead propagate it until Earth, Mars, and the cycler vehicle all return to their initial state in inertial space (approximately). The ballistic S1L1 cycler repeats inertially after 30 years (seven cycles of $4\frac{2}{7}$ years). The orbits of Earth and Mars also repeat approximately every 30 years (after 30 Earth revolutions and 16 Mars revolutions). Therefore, if we can propagate a two-synodic-period cycler for 30 years or more, then we can be fairly confident that it will propagate into the distant future (in the ephemeris model).

The problem of propagating a ballistic cycler for 30 years or more can be formulated as finding a solution to a system of N equations and N inequalities in $(N+2)$ unknowns. The $(N+2)$ unknowns are the encounter times $t_0, t_1, \ldots, t_{N+1}$ at the bodies, as illustrated in Fig. 4. For the remainder of this paper, we will call a sequence of encounter times $(t_0, t_1, \ldots, t_{N+1})$ an "itinerary." The type of transfer on each leg is known from the circular-coplanar analysis, so that the body encounter times uniquely determine the incoming and outgoing V_∞ at the N intermediate flybys. For the ith flyby to be ballistic, the magnitudes of the incoming V_∞ and the outgoing V_∞ must be equal:

$$V_{\infty\text{in},i}(t_{i-1}, t_i) - V_{\infty\text{out},i}(t_i, t_{i+1}) = 0 \qquad \text{for} \qquad i = 1, 2, \ldots, N \tag{1}$$

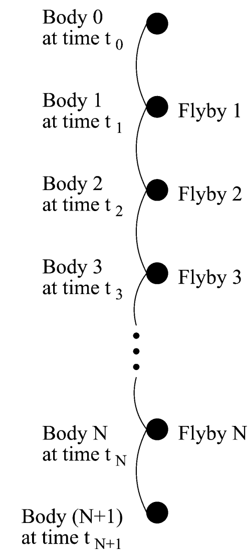


Fig. 4 $(N+2)$ bodies and the N flybys.

and the required flyby altitude h_i must be higher than 300 km (so that the cycler vehicle does not experience significant atmospheric drag):

$$h_i(t_{i-1}, t_i, t_{i+1}) - 300 \ge 0 \qquad \text{for} \qquad i = 1, 2, \ldots, N \tag{2}$$

Equations (1) and (2) can be neatly summarized using the vector notation

$$\boldsymbol{g}(\boldsymbol{t}) = \boldsymbol{0} \tag{3}$$

$$\boldsymbol{h}(\boldsymbol{t}) \ge \boldsymbol{0} \tag{4}$$

where $\boldsymbol{t} = (t_0, t_1, \ldots, t_{N+1})$ is an itinerary, Eq. (3) is the V_∞ constraint, and Eq. (4) is the altitude constraint. An itinerary $\boldsymbol{t}$ satisfying Eqs. (3) and (4) will be referred to as a "feasible itinerary" or "feasible solution." If none of the altitude constraints is active (i.e., if none of the flybys occurs at 300 km), then the solution to Eqs. (3) and (4) is not unique. In fact, because there are N equations in $(N+2)$ unknowns, we expect the set of solutions to have two degrees of freedom.

We used the software package SNOPT[27] (which stands for sparse nonlinear optimizer) to find a solution to Eqs. (3) and (4). Because SNOPT uses a quasi-Newton method to solve systems of equations and inequalities, it requires a good initial guess of the solution (i.e. the body-encounter times $\boldsymbol{t}$). At first, we tried to use the solution in the simplified (circular-coplanar) model of the solar system as our initial guess, but SNOPT could not converge from that guess to a solution in the ephemeris model.

Because the ephemeris model differs significantly from the simplified model, the solution to the simplified model is a bad initial guess for the solution to the ephemeris model. We decided to use a continuation approach to surmount this problem.The orbital elements of Earth and Mars were combined into a constant parameter vector, which we call $\boldsymbol{a}$. The system of equations and inequalities then has the form

$$\boldsymbol{g}(\boldsymbol{t}, \boldsymbol{a}) = \boldsymbol{0} \tag{5}$$

$$\boldsymbol{h}(\boldsymbol{t}, \boldsymbol{a}) \ge \boldsymbol{0} \tag{6}$$

For the simplified solar-system model, $\boldsymbol{a} = \boldsymbol{a}_{\text{simplified}}$, and for a more accurate solar-system model $\boldsymbol{a} = \boldsymbol{a}_{\text{accurate}}$. (The orbital elements in $\boldsymbol{a}_{\text{simplified}}$ and $\boldsymbol{a}_{\text{accurate}}$ are constants, and so the orbital elements of $\boldsymbol{a}_{\text{accurate}}$ do not represent the time-varying orbital elements of the ephemeris model.) We already have a solution when $\boldsymbol{a} = \boldsymbol{a}_{\text{simplified}}$, and we would like to find a solution when $\boldsymbol{a} = \boldsymbol{a}_{\text{accurate}}$. The solution when $\boldsymbol{a} = \boldsymbol{a}_{\text{accurate}}$ could then be used as an initial guess for the ephemeris model.

We can define a sequence of problems with $\boldsymbol{a}$ varying from $\boldsymbol{a}_{\text{simplified}}$ (whose solution is summarized in Table 1) to $\boldsymbol{a}_{\text{accurate}}$. By using the solution to the kth problem in the sequence as the initial guess for the $(k+1)$st problem in the sequence, we can ultimately arrive at the solution to the problem where $\boldsymbol{a} = \boldsymbol{a}_{\text{accurate}}$. The solution to the problem with $\boldsymbol{a} = \boldsymbol{a}_{\text{accurate}}$ is then used as an initial guess to the problem in which the positions and velocities of the planets are determined using JPL's DE405 ephemerides. We found that this continuation method worked very well, requiring only about 10 continuation iterations.

A 33-year, 24-body solution was found (in the ephemeris model) for all four vehicles (two outbound and two inbound) launching in the 2005–2007 time frame. (For the sake of brevity, the solution itineraries are not included here; they are available in Ref. 19.) None of the four solutions required any ΔV maneuvers, that is, they were all ballistic.

Because there are N equations in $(N+2)$ unknowns, we expect an infinite, two-dimensional set of solutions (provided that none of the flyby altitude constraints becomes active). Each solution has different characteristics. To determine the range of characteristics in a typical set of solutions, we found a large set of 27-body itineraries[19] for vehicle 2 (outbound). (We arbitrarily number the outbound vehicles 1 and 2, and the inbound vehicles 3 and 4.) Figure 5 shows the range of possible launch and arrival dates for the 2549 feasible solutions that were found. When launch dates earlier than 11 June 2005 were tried, there was either no solution or one of the flyby altitude constraints was violated. The top, bottom, and right boundaries on Fig. 5 occurred when flyby altitude constraints were violated. A surprising feature of the solutions shown in Fig. 5 is that the range of possible arrival dates does not depend on the launch date (and vice versa).

Even though there is a wide range of possible launch (and arrival) dates, we found that the middle dates (such as the dates at bodies 12–16 in a 27-body itinerary) are nearly the same. As shown in Fig. 6, the range of possible dates is wide near launch (at body 1) and near arrival (at body 27), but is nearly zero at the middle bodies. To explain why the middle dates do not vary much, let us consider the number of constraints that each time t_i must satisfy. The time t_0 only needs to satisfy one constraint, namely, the flyby V_∞ constraint at body 2. The time t_1 must satisfy two constraints (at bodies 2 and 3). The time t_2 must satisfy three constraints (at bodies 2–4). Three constraints are the most that any encounter time has to satisfy, no matter how many encounter times there are. Among the times that must satisfy three constraints, the ones in the middle can vary less because their neighbors are also highly constrained.

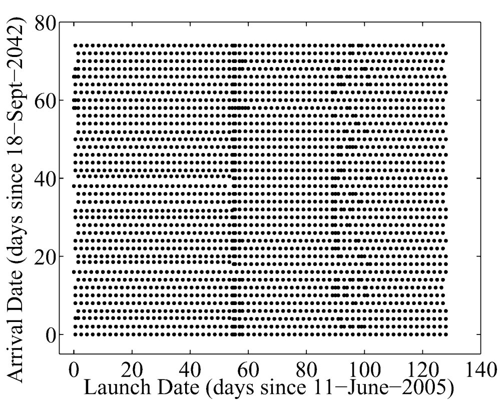


Fig. 5 Launch and arrival dates of feasible itineraries for vehicle 2 (outbound) on a ballistic S1L1 trajectory.

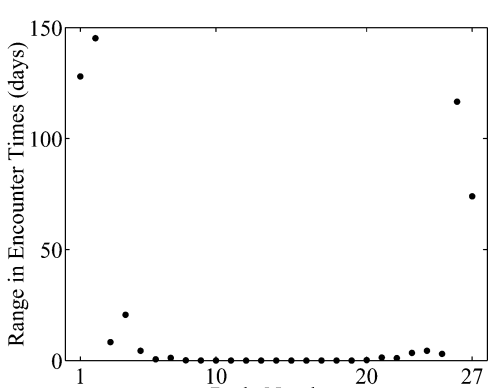


Fig. 6 Range of possible dates at each body.

If a cycler's itinerary is augmented to include more encounters, then one would expect the number of slightly varying middle times to increase, and that is exactly what happens. If one adds enough encounters, then a flyby altitude constraint violation occurs, and a feasible solution (itinerary) ceases to exist. Up to that point, one might wonder, Are the slightly varying middle times the only ones possible?

To analyze this question, suppose that the relative orientations of the orbits of Earth and Mars repeat inertially (in the ephemeris model) after a time T. (In the simplified model of the preceding section, $T = 15$ years.) We would expect T to be approximately equal to an integer multiple of the Earth–Mars synodic period S, that is, $T \approx mS$, where m is an integer. Because Earth and Mars repeat inertially after T years, one might look for a ballistic S1L1 cycler trajectory that also repeats after T years (or $2T$ years if m is odd). One would try to find the $(N+2)$ encounter times satisfying the usual N constraints, plus two new constraints: 1) the V_∞ at the initial time should equal the V_∞ at the final time, and 2) the final time should be the initial time plus T (or $2T$ if m is odd). A solution is not guaranteed to exist, but if one does exist it is unique, because there are $(N+2)$ independent equations in $(N+2)$ unknowns (by the rank theorem,[28] which holds even when the equations are nonlinear). We call the unique periodic solution the "ideal" cycler trajectory (or cycler).

We conjecture that the middle times of a long itinerary are the same as the times of the ideal cycler trajectory (and hence unique). To see why this is plausible, consider what happens as we add more encounters to the ends of a long itinerary. Eventually, the middle times of the long itinerary include all of the times of one period of the ideal cycler itinerary. Another explanation is that although the system of equations satisfied by a feasible long itinerary is not the same as the system of equations satisfied by the ideal cycler itinerary, both systems of equations enforce the same goal: finding an itinerary such that the cycler is ballistic for a very long time.

To find the itinerary of an ideal cycler, we find a 50-year itinerary and then keep the middle 30 years. That is, we assume that 10 years (or over two repetitions of the cycler) is sufficient for a cycler to converge to the ideal cycler.

As seen in Figs. 5 and 6, the launch period is about 128 days (or more than four months) long. The reason the launch date can vary is that it is not constrained to be on the ideal cycler (which stretches backwards in time). There might be only one ideal cycler, but there are many ways to get on to it (or off of it, for that matter). Because every cycler vehicle must be launched, a long launch period is useful. One could choose a nominal launch date, for example, to minimize the launch V_∞.

In some instances, a cycler trajectory that satisfies both Eqs. (5) and (6) does not exist, and we instead solve for the trajectory that requires the least ΔV (performed by the cycler in deep space: we ignore the ΔV required by the taxi vehicles for now). To solve for the minimum-ΔV trajectory, we begin by reducing the difference in V_∞ [i.e. we make Eq. (5) as close to zero as possible] while satisfying the altitude constraint [Eq. (6)]. Then, an initial guess for the time and location of a deep-space maneuver (DSM) is determined via Lawden's primer vector analysis.[29–31] For example, in Fig. 7 a DSM would be placed where the primer magnitude is largest [in this case around 7.5 years time of flight (TOF)]. An augmented trajectory (with the new DSM) is then optimized for minimum ΔV. Additional DSMs are added until a locally optimal trajectory is found. Figure 8 demonstrates an optimal trajectory, where the primer vector $\boldsymbol{P}$ satisfies the five conditions for optimality:

1) $\boldsymbol{P}$ and $(\mathrm{d}/\mathrm{d}t)\boldsymbol{P}$ are continuous.
2) $\boldsymbol{P}$ is aligned with $\boldsymbol{\Delta V}$ at impulse times.

Table 2 Itinerary for cycler vehicle 1 (outbound) in which only the cycler ΔV is minimized

Encounter	Date, mm/dd/yyyy	V_∞, km/s	Closest approach altitude, km	Leg TOF, days
Earth-1	10/14/2007	5.19	21,400	——
Mars-2	05/18/2008	2.98	11,200	216[a]
Earth-3	07/16/2010	7.02	24,800	790
Earth-4	01/09/2012	7.02	37,400	542
Mars-5	06/17/2012	5.89	9,800	160[a]
Earth-6	10/11/2014	5.31	25,500	846
Earth-7	03/29/2016	5.31	35,200	535
Mars-8	07/25/2016	7.86	9,700	118[a]
Earth-9	02/04/2019	3.99	22,600	924
Earth-10	07/18/2020	3.99	4,300	530
Mars-11	12/19/2020	4.36	5,100	154[a]
Earth-12	05/25/2023	6.09	20,000	887
Earth-13	11/13/2024	6.09	27,400	538
Mars-14	06/01/2025	3.71	16,100	200[a]
Earth-15	08/09/2027	6.73	25,900	799
Earth-16	01/31/2029	6.73	41,500	541
Mars-17	06/27/2029	6.61	16,600	148[a]
Earth-18	11/15/2031	4.43	31,800	871
Earth-19	04/30/2033	4.43	26,900	531
Mars-20	08/20/2033	7.34	10,700	113[a]
Earth-21	03/13/2036	4.25	22,000	936
Earth-22	08/25/2037	4.25	8,900	531

[a]Crew transfer time from Earth to Mars.

Table 3 Itinerary for cycler vehicle 2 (outbound) in which only the cycler ΔV is minimized

Encounter	Date, mm/dd/yyyy	V_∞, km/s	Closest approach altitude, km	Leg TOF, days
Earth-1	12/03/2009	6.87	31,100	——
Mars-2	06/07/2010	4.31	17,600	186[a]
Earth-3	08/24/2012	6.43	26,400	809
Earth-4	02/14/2014	6.43	41,500	539
Mars-5	07/03/2014	7.14	12,200	138[a]
Earth-6	12/09/2016	4.00	27,800	890
Earth-7	05/23/2018	4.00	19,900	530
Mars-8	09/15/2018	6.47	11,600	115[a]
Earth-9	04/06/2021	4.61	23,000	934
Earth-10	09/20/2022	4.61	14,800	532
Mars-11	05/01/2023	2.77	7,600	223[a]
Earth-12	07/02/2025	7.09	23,800	793
Earth-13	12/26/2026	7.09	35,200	542
Mars-14	06/14/2027	5.27	13,700	170[a]
Earth-15	09/21/2029	5.78	26,800	830
Earth-16	03/12/2031	5.78	39,200	537
Mars-17	07/15/2031	7.69	10,700	125[a]
Earth-18	01/16/2034	3.75	22,600	915
Earth-19	06/28/2035	3.75	10,000	529
Mars-20	11/14/2035	4.65	16,200	139[a]
Earth-21	05/08/2038	5.53	27,100	906
Earth-22	10/26/2039	5.53	23,900	536

[a]Crew transfer time from Earth to Mars.

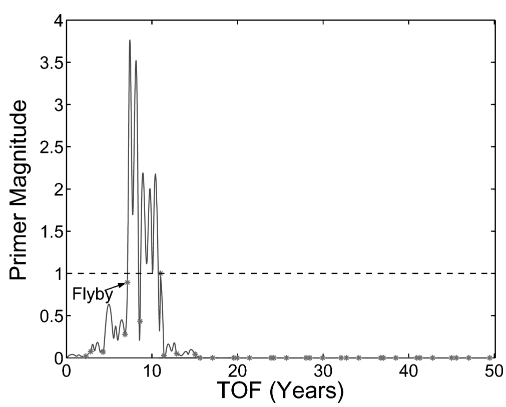


Fig. 7 Primer vector magnitude along suboptimal cycler trajectory.

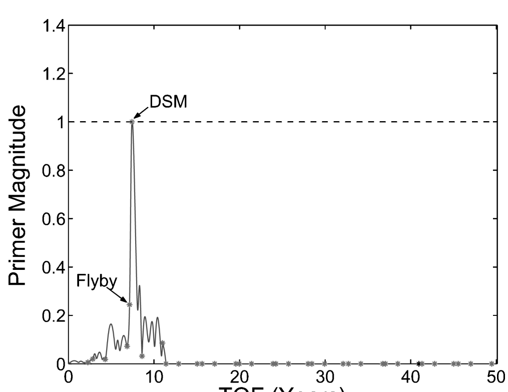


Fig. 8 Primer vector magnitude along optimal cycler trajectory.

Table 4 Itinerary for cycler vehicle 3 (inbound) in which only the cycler ΔV is minimized

Encounter	Date, mm/dd/yyyy	V_∞, km/s	Closest approach altitude, km	Leg TOF, days
Maneuver	04/04/2006	0.17[a]	——	——
Earth-1	06/07/2007	5.80	3,300	——
Mars-2	10/23/2009	5.83	300	869
Earth-3	03/26/2010	6.98	36,600	153[b]
Earth-4	09/19/2011	6.98	23,400	542
Mars-5	11/18/2013	3.26	11,900	791
Earth-6	06/15/2014	5.80	23,700	209[b]
Earth-7	12/04/2015	5.80	18,300	537
Mars-8	05/27/2018	4.59	7,000	905
Earth-9	10/15/2018	3.78	8,400	141[b]
Earth-10	03/27/2020	3.79	25,800	529
Mars-11	09/29/2022	7.72	12,500	916
Earth-12	01/28/2023	6.06	43,300	121[b]
Earth-13	07/19/2024	6.06	11,200	538
Mars-14	11/04/2026	5.58	2,100	838
Earth-15	04/17/2027	7.16	31,100	165[b]
Earth-16	10/11/2028	7.15	24,300	542
Mars-17	12/08/2030	2.76	9,700	788
Earth-18	07/21/2031	4.67	17,300	225[b]
Earth-19	01/03/2033	4.67	21,700	532
Mars-20	07/27/2035	6.34	9,300	934
Earth-21	11/19/2035	3.99	19,800	115[b]
Earth-22	05/01/2037	3.99	26,700	530

[a]Deep space ΔV. [b]Crew transfer time from Mars to Earth.

3) $|\boldsymbol{P}| = 1$ at impulse times.
4) $|\boldsymbol{P}| < 1$on coasting arcs separating impulses.
5) $(\mathrm{d}/\mathrm{d}t)|\boldsymbol{P}| = 0$ at impulses.

Tables 2–5 give 30-year itineraries for all four cycler vehicles in the ephemeris model (in which the total cycler ΔV is minimized over 50 years). The two outbound cyclers are ballistic (over the examined time span), but each inbound cycler requires a modest ΔV. Though not shown in Tables 2 and 3, the outbound trajectories also require a ΔV on the order of 10 m/s in the following 30-year cycle. So, although the S1L1 cycler is ballistic in the circular-coplanar model, it is only nearly ballistic in the ephemeris model, where the relative positions of Earth and Mars do not repeat precisely.

One interesting feature of the solutions in the ephemeris model (given in Tables 2–5) is that the flyby altitudes are generally lower than those of the solution in the circular-coplanar model. For example, the average Earth flyby altitude is 24,090 km, whereas the Earth flyby altitude in the circular-coplanar model is 31,809 km. Similarly, the average Mars flyby altitude is 9,950 km, whereas the Mars flyby altitude in the circular-coplanar model is effectively infinite (or at least outside of the Mars sphere of influence, which has

Table 5 Itinerary for cycler vehicle 4 (inbound) in which only the cycler ΔV is minimized

Encounter	Date, mm/dd/yyyy	V_∞, km/s	Closest approach altitude, km	Leg TOF, days
Earth-1	08/07/2009	6.24	19,100	——
Mars-2	11/06/2011	4.78	6,800	820
Earth-3	05/01/2012	7.06	29,800	177[a]
Earth-4	10/25/2013	7.06	24,800	542
Mars-5	01/08/2016	2.75	9,900	805
Earth-6	08/13/2016	4.15	13,600	218[a]
Earth-7	01/25/2018	4.15	22,500	530
Mars-8	08/20/2020	7.19	10,500	938
Earth-9	12/06/2020	4.68	31,200	108[a]
Earth-10	05/23/2022	4.68	16,000	532
Mars-11	10/16/2024	6.64	3,800	877
Earth-12	03/11/2025	6.74	40,800	146[a]
Earth-13	09/03/2026	6.74	22,400	541
Mars-14	11/11/2028	3.80	11,900	800
Earth-15	05/24/2029	6.74	27,000	194[a]
Earth-16	11/16/2030	6.74	9,000	541
Mars-17	04/13/2033	3.96	2,000	879
Earth-18	09/22/2033	3.86	6,800	162[a]
Earth-19	03/05/2035	3.86	24,600	529
Mars-20	09/17/2037	7.81	12,100	927
Earth-21	01/09/2038	5.52	39,800	114[a]
Earth-22	06/29/2039	5.52	14,400	536
Maneuver	12/01/2039	0.27[b]	——	——

[a]Crew transfer time from Mars to Earth. [b]Deep space ΔV.

Table 6 Itinerary for cycler vehicle 1 (outbound) in which total ΔV (taxi ΔV and cycler ΔV) is minimized

Encounter	Date, mm/dd/yyyy	V_∞, km/s	Closest approach altitude, km	Leg TOF, days
Earth-1	10/14/2007	5.16	21,300	——
Mars-2	05/31/2008	2.66	300	230[a]
Earth-3	07/24/2010	4.82	25,900	784
Maneuver	11/04/2010	0.66[b]	——	——
Earth-4	12/14/2011	4.33	26,200	508
Mars-5	06/16/2012	5.86	7,900	185[a]
Earth-6	10/11/2014	5.29	24,600	847
Earth-7	03/29/2016	5.29	35,200	535
Mars-8	07/25/2016	7.86	9,900	118[a]
Earth-9	02/04/2019	3.97	22,500	924
Earth-10	07/18/2020	3.97	4,600	530
Mars-11	12/20/2020	4.31	5,400	155[a]
Earth-12	05/26/2023	6.02	21,000	887
Maneuver	08/02/2023	0.11[b]	——	——
Earth-13	11/12/2024	5.85	29,400	536
Mars-14	06/21/2025	3.07	16,600	221[a]
Earth-15	08/23/2027	5.29	36,500	794
Maneuver	11/30/2027	0.81[b]	——	——
Earth-16	01/11/2029	4.61	31,700	507
Mars-17	06/27/2029	6.58	66,700	166[a]
Earth-18	11/15/2031	4.41	31,400	872
Earth-19	04/30/2033	4.41	26,700	531
Mars-20	08/20/2033	7.34	10,700	113[a]
Earth-21	03/13/2036	4.25	22,000	936
Earth-22	08/26/2037	4.25	9,000	531

[a]Crew transfer time from Earth to Mars. [b]Deep-space ΔV.

a radius of 577,400 km). The reason for the lower flyby altitudes (especially at Mars) is that the gravity-assist maneuvers must also change the orbital plane of the cycler vehicle. The Earth–Earth transfers all have an inclination close to zero, but the Earth–Mars and Mars–Earth transfers generally have a nonzero inclination because the inclination of Mars's orbit is about 1.85 deg.

Using Tables 2–5, one can construct example missions to Mars. To illustrate,suppose a group leaves Earth on vehicle 1 (outbound) on 9 January 2012. They would arrive at Mars 160 days later, on 17 June 2012. The next opportunity to leave Mars on an inbound cycler occurs 519 days (17 months) later, when vehicle 3 arrives on 18 November 2013. If the group returns at that time, they will have a 209-day transit back to Earth, where they would arrive on 15 June 2014. The entire mission would last 2.43 years.

There are 14 possible missions that can be constructed using Tables 2–5. Among those missions, Earth–Mars transfers last from 113 to 223 days (compared to 153 days in the circular-coplanar model), Mars stay times range from 516 to 706 days (compared to 605 days), Mars–Earth transfers last from 108 to 225 days (compared to 153 days), and total mission duration ranges from 2.41 to 2.56 years (compared to 2.49 years). Generally speaking, missions with long transit times tend to have short Mars stay times and vice versa.

Although the trajectories in Tables 2–5 were optimized to minimize the cycler vehicle ΔV, other cycler characteristics can influence a mission's cost. For the human exploration of Mars, we assume the mission begins with the crew departing Earth in a taxi vehicle that will rendezvous with the outbound cycler vehicle (which was launched previously) as it swings by Earth. After about five months in interplanetary space, the crew arrives at Mars, where they reach the surface via an aero-assisted descent (that requires a negligible ΔV from the taxi). After about 20 months on Mars, the crew again climbs into a taxi to rendezvous with the inbound cycler. The crew then returns to Earth where they land on the surface after atmospheric deceleration to conclude their 30-month mission. Thus, pertinent mission criteria include (but are not limited to) the cycler vehicle ΔV, the Earth and Mars departure V_∞, the transit TOF, the stay time on Mars, and the total mission duration. The arrival V_∞ are less of a mission driver than the departure V_∞ because the aeroshell weighs much less than the propellant required at departure.

To accommodate for the mass of the cycler vehicle and the taxi, we redefine the cost metric as the sum of the cycler and departure taxi ΔV, where we assume that the departure maneuver is performed at an altitude of 300 km by an upper stage. We choose equal weighting on the taxi and cycler ΔV for simplicity because the optimal weighting to minimize mission cost is dependent on several factors beyond the scope of this paper (e.g., the relative mass of the taxi and cycler vehicles). The optimal cycler itineraries that arise when we use this new metric are provided in Tables 6–9. For the outbound vehicles, the larger Earth departure V_∞ values are reduced by adding a few modest ΔV maneuvers. For example, the departure V_∞ of vehicle 1 at Earth-4 was reduced from 7.02 to 4.33 km/s by a ΔV of 0.66 km/s on the previous leg. Over all 14 missions, the average Earth departure V_∞ was reduced from 5.52 to 4.90 km/s, a reduction accomplished by increasing the DSM ΔV to an average of 0.11 km/s per synodic period per outbound cycler vehicle. Similarly, the average Mars departure V_∞ was reduced from 5.22 to 3.22 km/s by increasing the DSM ΔV to an average of 0.33 km/s per synodic period per inbound cycler vehicle. We also note that the inbound vehicles sometimes pass as close as possible to Mars (at the altitude constraint of 300 km).

One might wonder what the results would be if a mass-based cost metric were used. Possible mass-based metrics include the average initial mass in low Earth orbit (IMLEO) and the total mass launched into low-Earth orbit over the lifetime of the system. Unfortunately, to use mass as a cost metric, one must make several extra design decisions. For example, one must choose the number of passengers, the mass of the cycler vehicles, the mass of the taxi vehicles, the mass of repairs or replacements, the propulsion systems, and the lifetime of the system. Because each decision has several plausible options and because we do not want to bias the reader toward any particular architecture, we leave the use of mass-based metrics to future mission designers.

The crew transfer TOF on these cyclers can sometimes become relatively long (e.g., the 239-day TOF from Mars-8 to Earth-9 in Table 8). In the event that the interplanetary transfer time must be constrained (e.g., because of radiation or zero-gravity crew health concerns), the cycler trajectories still exist, but the ΔV and V_∞ requirements increase. Figures 9–11 show how the cycler

Table 7 Itinerary for cycler vehicle 2 (outbound) in which total ΔV (taxi ΔV and cycler ΔV) is minimized

Encounter	Date, mm/dd/yyyy	V_∞, km/s	Closest approach altitude, km	Leg TOF, days
Maneuver	08/31/2008	0.30[a]	——	——
Earth-1	11/28/2009	6.19	31,200	——
Mars-2	06/14/2010	3.94	14,800	199[b]
Earth-3	09/02/2012	5.52	33,100	811
Maneuver	12/04/2012	0.62[a]	——	——
Earth-4	01/31/2014	4.83	33,900	516
Mars-5	07/02/2014	7.12	25,700	152[b]
Earth-6	12/09/2016	4.00	27,800	891
Earth-7	05/22/2018	4.00	19,300	530
Mars-8	09/15/2018	6.50	11,300	116[b]
Earth-9	04/06/2021	4.65	22,900	934
Earth-10	09/20/2022	4.65	13,500	532
Mars-11	05/10/2023	2.64	7,000	233[b]
Earth-12	07/04/2025	6.86	27,800	786
Maneuver	09/19/2025	0.31[a]	——	——
Earth-13	12/21/2026	6.36	33,500	534
Mars-14	06/15/2027	5.18	12,500	177[b]
Earth-15	09/24/2029	5.53	29,100	832
Maneuver	12/16/2029	0.27[a]	——	——
Earth-16	03/06/2031	5.13	35,800	528
Mars-17	07/15/2031	7.69	14,400	131[b]
Earth-18	01/16/2034	3.75	22,600	916
Earth-19	06/14/2035	3.75	9,700	529
Mars-20	11/14/2035	4.65	16,200	139[b]
Earth-21	05/08/2038	5.53	27,100	907
Earth-22	10/26/2039	5.53	23,900	536

[a]Deep-space ΔV. [b]Crew transfer time from Earth to Mars.

Table 8 Itinerary for cycler vehicle 3 (inbound) in which total ΔV (taxi ΔV and cycler ΔV) is minimized

Encounter	Date, mm/dd/yyyy	V_∞, km/s	Closest approach altitude, km	Leg TOF, days
Earth-1	06/18/2007	4.91	50,000	——
Maneuver	01/19/2008	1.31[a]	——	——
Mars-2	09/05/2009	3.82	2,400	810
Earth-3	03/23/2010	6.92	65,600	200[b]
Earth-4	09/16/2011	6.92	26,100	542
Maneuver	05/07/2012	0.12[a]	——	——
Mars-5	10/29/2013	3.00	17,500	774
Earth-6	05/28/2014	7.84	29,600	210[b]
Earth-7	11/24/2015	7.84	34,600	545
Maneuver	07/13/2016	0.15[a]	——	——
Mars-8	01/16/2018	2.90	3,500	784
Earth-9	09/12/2018	4.32	27,500	239[b]
Earth-10	02/25/2020	4.32	52,300	531
Maneuver	09/08/2020	1.51[a]	——	——
Mars-11	07/25/2022	3.94	300	881
Earth-12	01/29/2023	5.07	70,700	189[b]
Earth-13	07/16/2024	5.07	23,300	534
Maneuver	02/08/2026	0.73[a]	——	——
Mars-14	09/06/2026	3.32	15,200	781
Earth-15	04/15/2027	7.03	49,500	221[b]
Earth-16	10/08/2028	7.03	25,000	542
Mars-17	11/24/2030	2.64	21,100	777
Earth-18	06/26/2031	6.77	25,900	214[b]
Earth-19	12/18/2032	6.77	40,500	541
Maneuver	07/18/2033	0.64[a]	——	——
Mars-20	04/04/2035	3.21	300	837
Earth-21	11/04/2035	3.36	41,400	214[b]
Earth-22	04/14/2037	3.36	14,500	527

[a]Deep-space ΔV. [b]Crew transfer time from Mars to Earth.

characteristics change as the maximum allowable TOF ranges from 120 to 270 days (four to nine months). To calculate these constrained-transfer-TOF trajectories, the trajectory with a maximum transfer TOF of 270 days (which we will call the "270-day TOF trajectory") served as the initial guess for the 260-day TOF trajectory, and so on (by 10-day steps) until the 120-day TOF trajectory was optimized. We then repeated the process in reverse (from 120- to 270-day TOF) to help ensure we had found the minimum-ΔV trajectories.

Each line on Figs. 9–11 corresponds to a different round-trip Earth–Mars–Earth mission. Each mission involves one outbound vehicle and one inbound vehicle. Vehicles 1 (outbound) and 3 (inbound) always work together, as do vehicles 2 and 4. Each line is labeled with the year in which the astronauts leave Earth on an outbound vehicle to go to Mars. For example, in Table 6 we find that vehicle 1 (outbound) leaves Earth to go to Mars on 14 December 2011. On Figs. 9–11, the line labeled "2011" refers to the entire Earth–Mars stay–Mars–Earth mission that begins in 2011. After a 185-day transit, the 2011 mission arrives at Mars on 16 June 2012. The astronauts stay at Mars for (exactly) 500 days until vehicle 2 (with itinerary given in Table 8) arrives on 29 October 2013 to take them back to Earth. After a 210-day transit, the astronauts arrive back at Earth on 28 May 2014. The total mission duration is $185 + 500 + 210 = 895$ days. (The just-described mission assumes a maximum transit TOF, or max. TOF, of 270 days.)

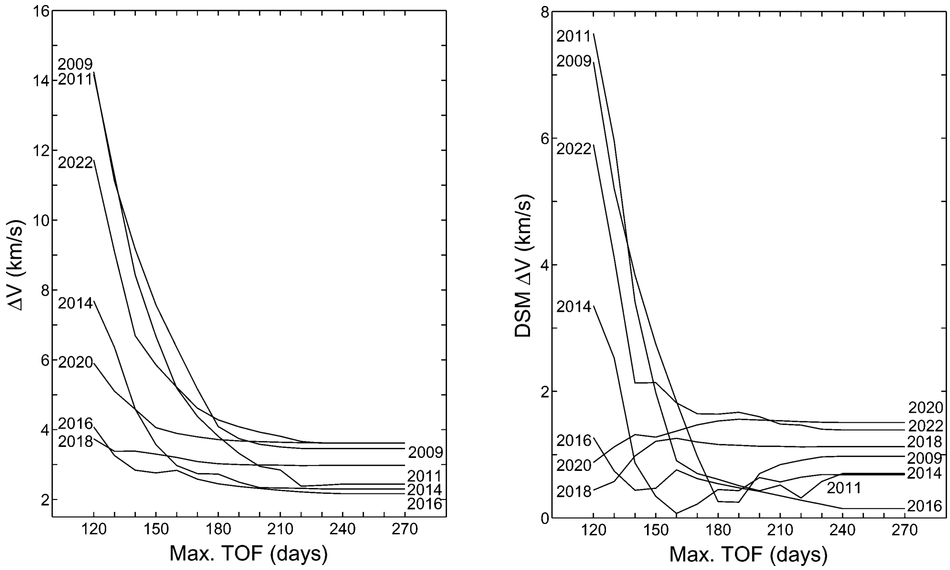
Only seven mission opportunities (or launch years) need to be considered because the mission characteristics repeat after that. The launch dates for missions 1 and 8 are 15 years apart. Consequently, Earth and Mars are in (approximately) the same inertial position at the beginning of mission 8 as they were at the beginning of mission 1. The only difference is that vehicles 2 and 4 will carry out mission 8 instead of vehicles 1 and 3 (or vice versa). (Each cycler vehicle returns to the same inertial position every 30 years.)

From Fig. 9b, we see that the minimum DSM ΔV for the 2011 mission is just under 1 km/s. We note that the DSM ΔV includes ΔV maneuvers performed by both the outbound and the inbound cycler vehicles (for the legs on which humans were being transported as well as the legs immediately before and after those legs).

Table 9 Itinerary for cycler vehicle 4 (inbound) in which total ΔV (taxi ΔV and cycler ΔV) is minimized

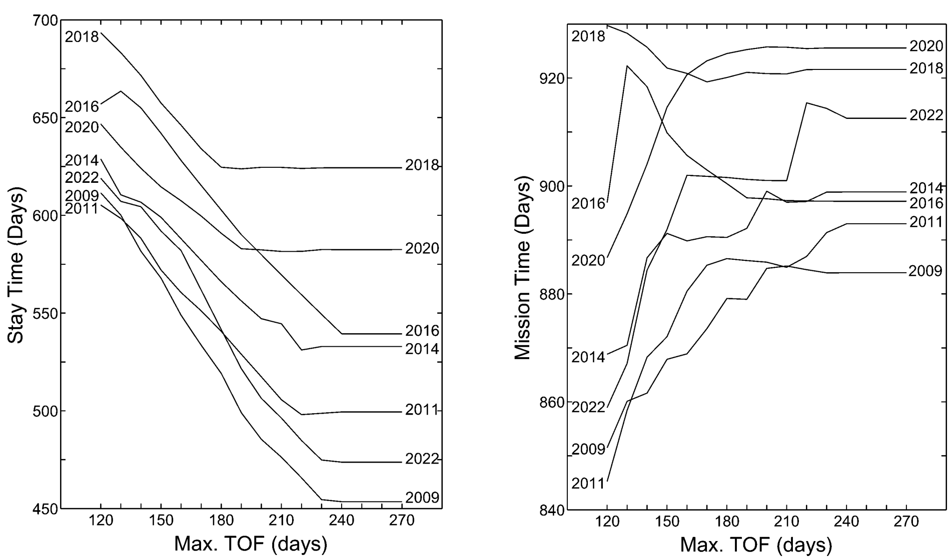
Encounter	Date, mm/dd/yyyy	V_∞, km/s	Closest approach altitude, km	Leg TOF, days
Earth-1	08/05/2009	5.70	24,800	——
Maneuver	02/10/2011	0.56[a]	——	——
Mars-2	09/10/2011	3.05	20,500	767
Earth-3	04/29/2012	6.93	40,300	231[b]
Earth-4	10/22/2013	6.93	25,900	542
Maneuver	08/28/2014	0.07[a]	——	——
Mars-5	12/15/2015	2.48	20,400	784
Earth-6	07/17/2016	5.90	25,200	215[b]
Earth-7	01/05/2018	5.90	35,900	537
Maneuver	07/16/2018	1.12[a]	——	——
Mars-8	05/31/2020	3.50	300	876
Earth-9	11/28/2020	3.97	57,800	182[b]
Earth-10	05/12/2022	3.97	44,800	530
Maneuver	12/19/2022	1.37[a]	——	——
Mars-11	08/25/2024	3.74	2,300	836
Earth-12	03/20/2025	5.36	66,100	207[b]
Earth-13	09/06/2026	5.36	22,300	535
Maneuver	03/12/2028	0.12[a]	——	——
Mars-14	09/24/2028	2.90	3,700	749
Earth-15	05/15/2029	7.56	32,800	233[b]
Earth-16	11/10/2030	7.56	30,300	544
Maneuver	07/24/2031	0.11[a]	——	——
Mars-17	12/19/2032	2.62	10,600	770
Earth-18	08/21/2033	4.80	22,400	245[b]
Earth-19	02/05/2035	4.80	40,000	533
Maneuver	08/26/2035	1.38[a]	——	——
Mars-20	07/13/2037	3.92	300	889
Earth-21	01/07/2038	4.79	69,100	178[b]
Earth-22	06/23/2039	4.79	24,000	533

[a]Deep-space ΔV. [b]Crew transfer time from Mars to Earth.



a) Total optimized ΔV (taxi and cycler vehicle) **b) DSM**

Fig. 9 ΔV over 15-year cycle.



a) Mars stay time **b) Total mission duration**

Fig. 10 Stay time and mission duration over 15-year cycle.

In Fig. 9a, we see that the minimum total ΔV for the 2011 mission is between 2 and 3 km/s. The total optimized ΔV includes 1) the taxi ΔV to transfer from a parabolic orbit of Earth to the outbound cycler vehicle, 2) the taxi ΔV to transfer from a parabolic orbit of Mars to the inbound cycler vehicle, and 3) the DSM ΔV. The ΔV required to take the taxi from the surface of Earth to low Earth orbit (LEO) (or the surface of Mars to LMO) is not included because it is constant and thus does not affect the ΔV minimization problem. Similarly, the ΔV required to take the taxi from LEO to a parabolic orbit of the Earth plus the ΔV required to take the taxi from LMO to a parabolic orbit of Mars is not included because it is constant (4.6 km/s). As transit TOF is constrained, the required total ΔV generally increases. For missions with short transit TOF, the 2009 and 2011 missions require the most ΔV, whereas the 2018 mission requires the least. If transit times of 230 days (or more) are considered acceptable, then the maximum total ΔV ever required by a mission is less than 4 km/s.

Figure 10a shows that as one shortens the transit TOF, one lengthens the stay time on Mars. The 2018 mission has the longest stay times. If the maximum transit time is 120 days, then the 2018 mission crew stays at Mars for almost 700 days (or 1.92 years). On the other end of the spectrum, the crew of the 2009 mission could be at Mars for as few as 450 days (or 1.23 years). (The stay time in the circular-coplanar model is 1.66 years.)

In Fig. 10b, we see that as one allows for longer transit times the total mission duration usually becomes longer (with the 2018 and 2016 missions being notable exceptions). Total mission duration ranges from 845 to 930 days (2.31 to 2.55 years). (In the circular-coplanar model, the total mission duration is 2.49 years.)

To interpret Fig. 11, we note that the Earth departure V_∞ matters only for the outbound cycler because a taxi vehicle has to rendezvous with it. Similarly, the Mars arrival V_∞ matters only for the outbound cycler, the Mars departure V_∞ matters only for the inbound cycler, and the Earth arrival V_∞ matters only for the inbound cycler. Consequently, Figs. 11a and 11b refer to the outbound cycler vehicle, and Figs. 11c and 11d refer to the inbound cycler vehicle. As expected, if one shortens the acceptable transit time, the V_∞ usually increases. The V_∞ at Earth ranges from 4 to 12 km/s, and the V_∞ at Mars ranges from 2.5 to 9.5 km/s. The 2018 missions tend

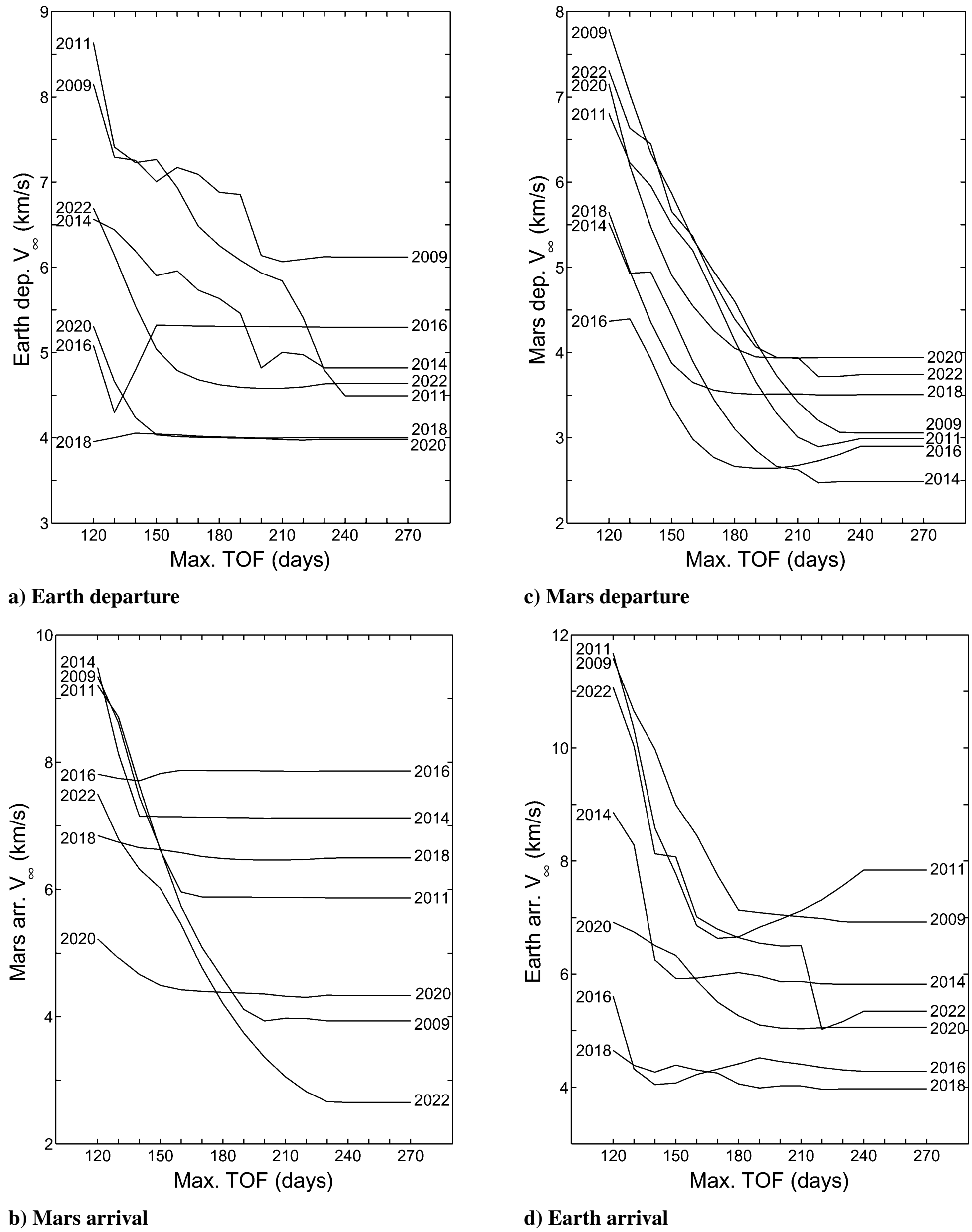


Fig. 11 V_∞ over 15-year cycle.

to have one of the lowest V_∞ (but they also have the longest total mission duration).

Conclusions

Implementation of Earth–Mars cycler trajectories could be a key enabling factor in the human exploration and development of Mars. In recent years, many new cycler trajectories have been discovered. Among these, the ballistic S1L1 cycler has characteristics that make it attractive for a human transportation system. We analyzed this new cycler in a circular-coplanar model and in a more accurate, ephemeris model of the solar system. Minimum-ΔV S1L1 cycler itineraries were found for 14 Earth–Mars synodic periods (about 30 years), after which the orbits of Earth, Mars, and the cycler approximately repeat in inertial space. Also, key characteristics of the S1L1 cycler were plotted as a function of maximum allowable TOF between Earth and Mars. These data could be particularly useful for preliminary design studies of human missions to Mars.

Acknowledgments

This research has been supported in part by an award from the Purdue Research Foundation (first author) and by a National Defense Science and Engineering Graduate Fellowship (second author).

References

[1]Bishop, R. H., Byrnes, D. V., Newman, D. J., Carr, C. E., and Aldrin, B., "Earth-Mars Transportation Opportunities: Promising Options for Interplanetary Transportation," American Astronautical Society, AAS Paper 00-255, March 2000.

[2]Aldrin, B., Byrnes, D. V., Jones, R., and Davis, H., "Evolutionary Space Transportation Plan for Mars Cycling Concepts," AIAA Paper 2001-4677, Aug. 2001.

[3]Landau, D. F., and Longuski, J. M., "A Reassessment of Trajectory Options for Human Missions to Mars," AIAA Paper 2004-5095, Aug. 2004.

[4]Niehoff, J., Friedlander, A., and McAdams, J., "Earth–Mars Transportation Cycler Concepts," International Astronautical Congress, IAF Paper 91-483, Oct. 1991.

[5]Penzo, P. A., and Nock, K. T., "Earth–Mars Transportation Using Stop-Over Cyclers," AIAA Paper 2002-4424, Aug. 2002.

[6]Rall, C. S., "Free-Fall Periodic Orbits Connecting Earth and Mars," Sc.D. Thesis, Dept. of Aeronautics and Astronautics, Massachusetts Inst. of Technology, Cambridge, MA, Oct. 1969.

[7]Rall, C. S., and Hollister, W. M., "Free-Fall Periodic Orbits Connecting Earth and Mars," AIAA Paper 71-92, Jan. 1971.

[8]Aldrin, E. E., "Cyclic Trajectory Concepts," SAIC presentation to the Interplanetary Rapid Transit Study Meeting, Jet Propulsion Lab., Oct. 1985.

[9]Niehoff, J., "Manned Mars Mission Design," Steps to Mars, Joint AIAA/Planetary Society Conf., National Academy of Sciences, Pasadena, CA, 1985.

[10]Niehoff, J., "Integrated Mars Unmanned Surface Exploration (IMUSE), A New Strategy for the Intensive Science Exploration of Mars," Space Science Board, Woods Hole, MA, July 1985, pp. 1–10.

[11]Niehoff, J., "Pathways to Mars: New Trajectory Opportunities," American Astronautical Society, Paper 86-172, July 1986.

[12]Byrnes, D. V., Longuski, J. M., and Aldrin, B., "Cycler Orbit Between Earth and Mars," *Journal of Spacecraft and Rockets*, Vol. 30. No. 3, 1993, pp. 334–336.

[13]Friedlander, A. L., Niehoff, J. C., Byrnes, D. V., and Longuski, J. M., "Circulating Transportation Orbits Between Earth and Mars," AIAA Paper 86-2009, Aug. 1986.

[14]McConaghy, T. T., Longuski, J. M., and Byrnes, D. V., "Analysis of a Broad Class of Earth-Mars Cycler Trajectories," AIAA Paper 2002-4420, Aug. 2002.

[15]Byrnes, D. V., McConaghy, T. T., and Longuski, J. M., "Analysis of Various Two Synodic Period Earth-Mars Cycler Trajectories," AIAA Paper 2002-4423, Aug. 2002.

[16]Chen, K. J., McConaghy, T. T., Okutsu, M., and Longuski, J. M., "A Low-Thrust Version of the Aldrin Cycler," AIAA Paper 2002-4421, Aug. 2002.

[17]Chen, K. J., Landau, D. F., McConaghy, T. T., Okutsu, M., Longuski, J. M., and Aldrin, B., "Preliminary Analysis and Design of Powered Earth-Mars Cycling Trajectories," AIAA Paper 2002-4422, Aug. 2002.

[18]Chen, K. J., McConaghy, T. T., Landau, D. F., Longuski, J. M., and Aldrin, B., "Powered Earth–Mars Cycler with Three-Synodic-Period Repeat Time," *Journal of Spacecraft and Rockets*, Vol. 42, No. 5, 2005, pp. 921–927.

[19]McConaghy, T. T., Yam, C. H., Landau, D. F., and Longuski, J. M., "Two-Synodic Period Earth–Mars Cyclers with Intermediate Earth Encounter," American Astronautical Society, AAS Paper 03-509, Aug. 2003.

[20]McConaghy, T. T., Longuski, J. M., and Byrnes, D. V., "Analysis of a Class of Earth–Mars Cycler Trajectories," *Journal of Spacecraft and Rockets*, Vol. 41, No. 4, 2004, pp. 622–628.

[21]Rauwolf, G. A., Friedlander, A. L., and Nock, K. T., "A Mars Cycler Architecture Utilizing Low-Thrust Propulsion," AIAA Paper 2002-5046, Aug. 2002.

[22]Russell, R. P., and Ocampo, C. A., "Geometric Analysis of Free-Return Trajectories Following a Gravity-Assist Flyby," *Journal of Spacecraft and Rockets*, Vol. 42, No. 1, 2005, pp. 138–152.

[23]Russell, R. P., and Ocampo, C. A., "Global Search for Idealized Free-Return Earth–Mars Cyclers," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 2, 2005, pp. 194–208.

[24]Russell, R. P., and Ocampo, C. A.,"Systematic Method for Constructing Earth–Mars Cyclers Using Free-Return Trajectories," *Journal of Guidance, Control, and Dynamics*, Vol. 27, No. 3, 2004, pp. 321–335.

[25]McConaghy, T. T., Russell, R. P., and Longuski, J. M., "Toward a Standard Nomenclature for Earth–Mars Cycler Trajectories," *Journal of Spacecraft and Rockets*, Vol. 42, No. 4, 2005, pp. 694–698.

[26]Prussing, J. E., and Conway, B. A., *Orbital Mechanics*, Oxford Univ. Press, New York, 1993, pp. 62–80.

[27]Gill, P. E., Murray, W., and Saunders, M. A., "SNOPT: An SQP Algorithm for Large-Scale Constrained Optimization," *SIAM Journal on Optimization*, Vol. 12, No. 4, 2002, pp. 979–1006.

[28]Jones, A., Gray, A., and Hutton, R., *Manifolds and Mechanics*, Cambridge Univ. Press, Cambridge, England, U.K., 1987, pp. 33–35.

[29]Lawden, D. F., *Optimal Trajectories for Space Navigation*, Butterworths, London, 1963, pp. 54–69.

[30]Jezewski, D. J., "Primer Vector Theory and Applications," NASA TR R-454, Nov. 1975.

[31]Sauer, C. G., "Optimization of Interplanetary Trajectories with Unpowered Planetary Swingbys," American Astronautical Society, AAS Paper 87-424, Aug. 1987.

D. Spencer
Associate Editor